



# Università degli Studi di Pavia

## Dipartimento di Ingegneria Civile e Architettura

### The elastica arm balance, the torsional gun, and the self-encapsulation, or dripping, of an elastic rod

The problem of an elastic rod deforming in a plane, the so-called “planar elastica”, has a long history, rooting to Jacob Bernoulli (1654-1705), Daniel Bernoulli (1700-1782), Leonhard Euler (1707-1783), and Pieter van Musschenbroek (1692-1761), but is still actual and rich of applications, sometimes unexpected. The elastica has attracted a great interest in the past and has involved contributions from first-class scientists, including Kirchhoff, Love, and Born. The research on the elastica marked the initiation of the calculus of variations and promoted the development of the theory of elliptic functions. Nowadays the elastica represents a useful introduction to the theory of nonlinear bifurcation and stability, but is also an important tool in the field of soft robotics and in the design of compliant mechanisms. Moreover, the elastica can be effectively used to explain snake or fish locomotion and to design snake-like robots.

In this talk, the theory of a planar, nonlinear elastic rod developed by Euler is used to present new phenomena in which nonlinearities (related to the fact that equilibrium of the rod is reached at large displacements) play a fundamental role. These new phenomena are all connected to the concept of configurational forces.

These forces occur in all situations where a solid body can change its configuration through a release of elastic energy. Examples of these forces are the Peach-Koehler interactions between dislocations or the forces acting on a phase boundary during phase transformations. Speaking of elastic rods, is it possible to find configurational forces acting on these? Theoretical and experimental proofs of the existence of these forces have been recently given [1]. Once discovered, we have been able to design systems exhibiting various configurational forces. These are shown to deeply influence stability [2] and have inspired us the design of a new type of scale, in which both equilibrium (as in a rigid arm balance) and deformation (as in a spring balance) determine the solution of a highly nonlinear system that can be calculated and realized to measure weight [3].

Configurational forces can also be induced by torsion, a concept which has inspired to us the idea of a new type of torsional actuator, nicknamed 'torsional gun' [4].

Finally, since the Euler's differential equation of the elastica governs an oscillating pendulum, a buckling rod, and a pendant drop, we pose the problem of the dripping of an elastic rod, namely: can an elastic rod subject to a transversal force self-encapsulate and take the shape of a drop? This problem can be analytically solved and experimentally attacked.

#### References:

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- [2] D. Bigoni, F. Bosi, F. Dal Corso and D. Misseroni, Instability of a penetrating blade. *J. Mech. Phys. Solids*, 2014, 64, 411-425
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- [4] D. Bigoni, F. Dal Corso, D. Misseroni and F. Bosi, Torsional locomotion. *Proc. Royal Soc. A*, 2014, 20140599

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